

Example 3:

$$\lambda = \text{arrival rate} = 5 \text{ students/hour}$$

$$\mu = \text{service rate} = 1 \text{ student/10 minutes} = 6 \text{ students/hour}$$

$$\rho = \frac{\lambda}{\mu} = \frac{5 \text{ students/hour}}{6 \text{ students/hour}} = \frac{5}{6}$$

$$L = \frac{\lambda}{\mu - \lambda} \text{ (for an M/M/1 system)} = \frac{5}{6 - 5} = 5$$

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{5^2}{6(6 - 5)} = \frac{25}{6} = 4\frac{1}{6}$$

- % of time student can walk in without waiting = % of time system has no customers = P_0

$$P_0 \text{ for an M/M/1 system} = 1 - \rho$$

$$P_0 = 1 - \rho = 1 - \frac{5}{6} = \frac{1}{6}$$

- Probability of having to stand is $P_n | n > 4$

$$\begin{aligned} P_n | n > 4 &= \sum_{n=5}^{\infty} P_n = 1 - \sum_{n=0}^4 P_n = 1 - (P_0 + P_1 + P_2 + P_3 + P_4) \\ &= 1 - ((1-\rho) + (1-\rho)\rho + (1-\rho)\rho^2 + (1-\rho)\rho^3 + (1-\rho)\rho^4) \\ &= 1 - (1-\rho)[1 + \rho + \rho^2 + \rho^3 + \rho^4] \\ &= 1 - \left(\frac{1}{6}\right)\left(1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \dots + \left(\frac{5}{6}\right)^4\right) \\ &\approx .4018 \end{aligned}$$

$$W_q = \frac{\rho}{\mu - \lambda} = \frac{5/6}{6 - 5} = \frac{5}{6} \quad (\text{ugh! That's a long wait. Hope it is worth it})$$

$$W = \frac{1}{\mu - \lambda} = \frac{1}{6 - 5} = 1 \quad (\text{Makes sense! 50 minutes waiting + 10 minutes w/ Prof Nace})$$